Power Divider

Sairaj Dhople

Department of Electrical and Computer Engineering,
University of Minnesota, Minneapolis, MN, USA

Funding Acknowledgments
NSF and MiNnesota's Discover, Research and InnoVation Economy

January 28, 2016

Acknowledgment

Yu (Christine) Chen
Dept. of ECE, The University of British Columbia, Vancouver, Canada

A Note

Paper to appear in the IEEE Transactions on Power Systems

Power Divider

Yu Christine Chen, *Member, IEEE* and Sairaj V. Dhople, *Member, IEEE*

***Abstract*—This paper presents analytical closed-form expressions that uncover the contributions of nodal active- and reactive-power injections to the active- and reactive-power flows on transmission lines in an AC electrical network. Paying due homage to current- and voltage-divider laws that are similar in spirit, we baptize these as the *power divider* laws. Derived from a circuit-theoretic examination of AC power-flow expressions, the constitution of the power divider laws reflects the topology and voltage profile of the network. We demonstrate the utility of the power divider laws to the analysis of power networks by highlighting applications to transmission-network allocation, transmission-loss allocation, and identifying feasible injections while respecting line active-power flow set points.**

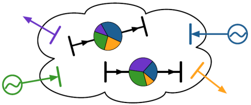


Fig. 1: The power divider laws: mapping injections to flows.

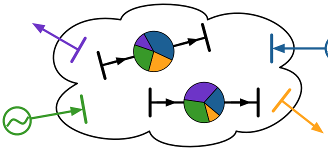
compared to state-of-the-art approaches. For instance, with regard to the task on identifying feasible injections, using

Outline

- Background
- Current Divider
- Power Divider
- Special Cases
- Applications

Key Question

How do real & reactive power injections map to real & reactive flows?



Why is this a hard problem?

- Nonlinear constant-power constraints
- Operating point dependence

Why would it be useful?

- Transmission-network cost allocation
- Power tracing

Previous Work

Previous approaches are numerical or rely on linear approximations

- Numerical methods [Kirschen ‘97, Bialek ‘98, Wollenberg ‘01]
- Utilizing current flows as proxies for power flows [Conejo ‘07]
- Generation shift distribution factors [Rudnick ‘95, Silva ‘13]

Previous Work

Previous approaches are numerical or rely on linear approximations

- Numerical methods [Kirschen '97, Bialek '98, Wollenberg '01]
- Utilizing current flows as proxies for power flows [Conejo '07]
- Generation shift distribution factors [Rudnick '95, Silva '13]

Contributions over methods in the literature

- Analytical, closed-form expressions
- Valid over entire operating regime
- Agnostic to location or specification of slack bus

Power System Model & Notation

- Connected system with N buses, Admittance matrix Y
- Complex-power bus injections: $S = [S_1, \ldots, S_N]^{\mathrm{T}}$, $S_\ell = P_\ell + \mathrm{j}Q_\ell$
- Voltage phasor, V: $|V| = [|V|_1, \ldots, |V|_N]^{\mathrm{T}}$; $\theta = [\theta_1, \ldots, \theta_N]^{\mathrm{T}}$
- Flow on line (m, n): $I_{(m,n)}$, $P_{(m,n)}$, $Q_{(m,n)}$
- Useful notation: $\theta^m = \theta_m \mathbf{1}_N - \theta$
- Useful notation:

$$\operatorname{diag}\left(\frac{\cos\theta^m}{|V|}\right) = \begin{bmatrix} \frac{1}{|V|_1} & 0 & 0 \\ 0 & \frac{\cos(\theta_1 - \theta_2)}{|V|_2} & 0 \\ 0 & 0 & \frac{\cos(\theta_1 - \theta_3)}{|V|_3} \end{bmatrix}, \quad m = 1, N = 3$$

Current Divider

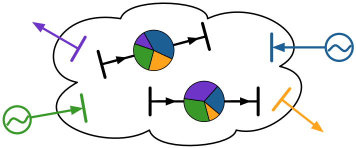
Contributions of current injections, $I = [I_1, I_2, \ldots, I_N]^{\mathrm{T}}$, to the current flow on line (m,n), $I_{(m,n)}$

$$\begin{aligned} I_{(m,n)} &= y_{mn}(V_m - V_n) + y_m V_m \\ &= \left(y_{mn} e_{mn}^{\mathrm{T}} + y_m e_m^{\mathrm{T}} \right) V \\ &= \left(y_{mn} e_{mn}^{\mathrm{T}} + y_m e_m^{\mathrm{T}} \right) Y^{-1} I \\ &= (\alpha_{(m,n)} + \mathrm{j}\beta_{(m,n)})^{\mathrm{T}} I. \end{aligned}$$

- Current divider only depends on network and not operating point
- $\alpha_{(m,n)}$, $\beta_{(m,n)} \in \mathbb{R}^N$: topology and impedances of network

Power Divider

How do real & reactive power injections map to real & reactive flows?



Contributions of real-power injections, $P = [P_1, P_2, \ldots, P_N]^{\mathrm{T}}$, and reactive-power injections $Q = [Q_1, Q_2, \ldots, Q_N]^{\mathrm{T}}$ to

- The real-power flow on line (m, n), $P_{(m,n)}$
- The reactive-power flow on line (m, n), $Q_{(m,n)}$

Power Divider

Contributions of nodal injections to real- and reactive-power flow on (m,n)

$$P_{(m,n)} = |V|_m \left(u_{(m,n)}^{\mathrm{T}} P - v_{(m,n)}^{\mathrm{T}} Q \right), \tag{1}$$

$$Q_{(m,n)} = |V|_m \left(u_{(m,n)}^{\mathrm{T}} Q + v_{(m,n)}^{\mathrm{T}} P \right). \tag{2}$$

In (1)–(2), $u_{(m,n)}, v_{(m,n)} \in \mathbb{R}^N$ are given by

$$u_{(m,n)} = \mathrm{diag}\ \frac{\cos\theta^m}{|V|}\ \alpha_{(m,n)} + \mathrm{diag}\ \frac{\sin\theta^m}{|V|}\ \beta_{(m,n)}, \tag{3}$$

$$v_{(m,n)} = \mathrm{diag}\ \frac{\sin\theta^m}{|V|}\ \alpha_{(m,n)} - \mathrm{diag}\ \frac{\cos\theta^m}{|V|}\ \beta_{(m,n)}. \tag{4}$$

Power Divider

Contributions of nodal injections to real- and reactive-power flow on (m,n)

$$P_{(m,n)} = |V|_m \Big(u_{(m,n)}^{\mathrm{T}} P - v_{(m,n)}^{\mathrm{T}} Q \quad , \tag{1}$$

$$Q_{(m,n)} = |V|_m \Big(u_{(m,n)}^{\mathrm{T}} Q + v_{(m,n)}^{\mathrm{T}} P \quad . \tag{2}$$

In (1)–(2), $u_{(m,n)}, v_{(m,n)} \in \mathbb{R}^N$ are given by

$$u_{(m,n)} = \operatorname{diag} \quad \frac{\cos \theta^m}{|V|} \quad \alpha_{(m,n)} + \operatorname{diag} \quad \frac{\sin \theta^m}{|V|} \quad \beta_{(m,n)}, \tag{3}$$

$$v_{(m,n)} = \operatorname{diag} \quad \frac{\sin \theta^m}{|V|} \quad \alpha_{(m,n)} - \operatorname{diag} \quad \frac{\cos \theta^m}{|V|} \quad \beta_{(m,n)}. \tag{4}$$

Power Divider

Contributions of nodal injections to real- and reactive-power flow on (m, n)

$$P_{(m,n)} = |V|_m \left(u_{(m,n)}^{\mathrm{T}} P - v_{(m,n)}^{\mathrm{T}} Q \right. \quad , \tag{1}$$

$$Q_{(m,n)} = |V|_m \left(u_{(m,n)}^{\mathrm{T}} Q + v_{(m,n)}^{\mathrm{T}} P \right. \quad . \tag{2}$$

In (1)–(2), $u_{(m,n)}, v_{(m,n)} \in \mathbb{R}^N$ are given by

$$u_{(m,n)} = \mathrm{diag} \quad \frac{\cos \theta^m}{|V|} \quad \alpha_{(m,n)} + \mathrm{diag} \quad \frac{\sin \theta^m}{|V|} \quad \beta_{(m,n)}, \tag{3}$$

$$v_{(m,n)} = \mathrm{diag} \quad \frac{\sin \theta^m}{|V|} \quad \alpha_{(m,n)} - \mathrm{diag} \quad \frac{\cos \theta^m}{|V|} \quad \beta_{(m,n)}. \tag{4}$$

Power Divider

Contributions of nodal injections to real- and reactive-power flow on (m, n)

$$P_{(m,n)} = |V|_m \left(u_{(m,n)}^{\mathrm{T}} P - v_{(m,n)}^{\mathrm{T}} Q \right. \quad , \tag{1}$$

$$Q_{(m,n)} = |V|_m \left(u_{(m,n)}^{\mathrm{T}} Q + v_{(m,n)}^{\mathrm{T}} P \right. \quad . \tag{2}$$

From the expressions, we see that the flows are

- Linear functions of real- and reactive-power injections: P, Q
- Nonlinear functions of the voltage profile: θ, $|V|$
- Embed network-topology information: $\alpha_{(m,n)}$, $\beta_{(m,n)}$

Minimal Realization

Contributions of nodal injections to real- and reactive-power flow on (m,n)

$$P_{(m,n)} = |V|_m \Big(u_{(m,n)}^{\mathrm{T}} P - v_{(m,n)}^{\mathrm{T}} Q \quad , \tag{1}$$

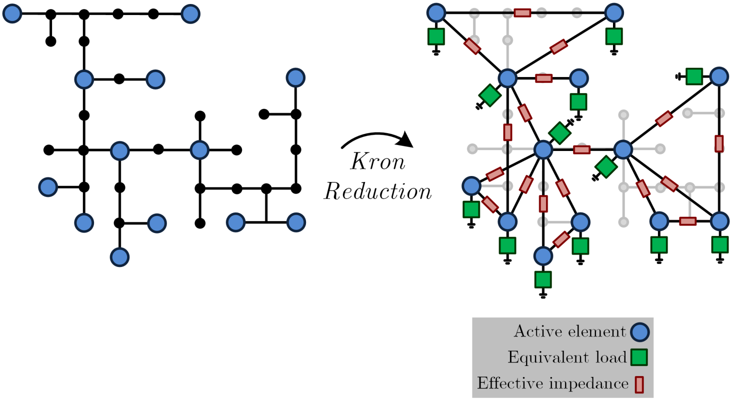
$$Q_{(m,n)} = |V|_m \Big(u_{(m,n)}^{\mathrm{T}} Q + v_{(m,n)}^{\mathrm{T}} P \quad . \tag{2}$$

In (1)–(2), $u_{(m,n)}, v_{(m,n)} \in \mathbb{R}^N$ are given by

$$u_{(m,n)} = \mathrm{diag} \quad \frac{\cos\theta^m}{|V|} \quad \alpha_{(m,n)} + \mathrm{diag} \quad \frac{\sin\theta^m}{|V|} \quad \beta_{(m,n)},$$
$$v_{(m,n)} = \mathrm{diag} \quad \frac{\sin\theta^m}{|V|} \quad \alpha_{(m,n)} - \mathrm{diag} \quad \frac{\cos\theta^m}{|V|} \quad \beta_{(m,n)}.$$

This suggests that voltage information from **all** buses is required

Minimal Realization



Kron reduction: Only generator- and load-bus information is needed

Some Special Cases

- Lossless networks: $Y = \mathrm{j}B$, $\beta_{(m,n)} = \mathbf{0}_N$

$$P_{(m,n)} = |V|_m \alpha_{(m,n)}^{\mathrm{T}} \left(\mathrm{diag}\left(\frac{\cos\theta^m}{|V|} \right) P - \mathrm{diag}\left(\frac{\sin\theta^m}{|V|} \right) Q \right).$$

- Small angle differences: $\theta_i - \theta_k \approx 0, \ \forall i, k$

$$P_{(m,n)} = |V|_m \alpha_{(m,n)}^{\mathrm{T}} \left(\mathrm{diag}\left(\frac{\mathbf{1}_N}{|V|} \right) P - \mathrm{diag}\left(\frac{\theta^m}{|V|} \right) Q \right).$$

- Uniform (unity) voltage profile: $|V|_k = 1, \ \forall k$

$$P_{(m,n)} = \alpha_{(m,n)}^{\mathrm{T}} \left(P - \mathrm{diag}\left(\theta^m\right) Q \right).$$

- Decoupling: $P >> \mathrm{diag}\left(\theta^m\right) Q$

$$P_{(m,n)} = \alpha_{(m,n)}^{\mathrm{T}} P.$$

Some Special Cases

- Lossless networks: $Y = \mathrm{j}B$, $\beta_{(m,n)} = \mathbf{0}_N$

$$P_{(m,n)} = |V|_m \alpha_{(m,n)}^{\mathrm{T}} \left(\mathrm{diag}\left(\frac{\cos\theta^m}{|V|} \right) P - \mathrm{diag}\left(\frac{\sin\theta^m}{|V|} \right) Q \right).$$

- Small angle differences: $\theta_i - \theta_k \approx 0, \ \forall i, k$

$$P_{(m,n)} = |V|_m \alpha_{(m,n)}^{\mathrm{T}} \left(\mathrm{diag}\left(\frac{\mathbf{1}_N}{|V|} \right) P - \mathrm{diag}\left(\frac{\theta^m}{|V|} \right) Q \right).$$

- Uniform (unity) voltage profile: $|V|_k = 1, \ \forall k$

$$P_{(m,n)} = \alpha_{(m,n)}^{\mathrm{T}} \left(P - \mathrm{diag}\left(\theta^m\right) Q \right).$$

- Decoupling: $P >> \mathrm{diag}\left(\theta^m\right) Q$

$$P_{(m,n)} = \alpha_{(m,n)}^{\mathrm{T}} P.$$

Decoupled Power Flow: Connection to Current Divider

- **Real-power** flow on (m, n) as a function of **real-power** injections

$$P_{(m,n)} = \alpha_{(m,n)}^{\mathrm{T}} P \tag{1}$$

for the case when we consider: i) lossless networks, ii) vanishingly small angle differences, and iii) uniform (unity) voltage profile

- **Current** flow on (m, n) as a function of **current** injections

$$I_{(m,n)} = \alpha_{(m,n)}^{\mathrm{T}} I \tag{2}$$

for the case when we consider lossless networks

Decoupled Power Flow: Connection to Current Divider

- **Real-power** flow on (m, n) as a function of **real-power** injections

$$P_{(m,n)} = \alpha_{(m,n)}^{\mathrm{T}} P \tag{1}$$

  for the case when we consider: i) lossless networks, ii) vanishingly small angle differences, and iii) uniform (unity) voltage profile

- **Current** flow on (m, n) as a function of **current** injections

$$I_{(m,n)} = \alpha_{(m,n)}^{\mathrm{T}} I \tag{2}$$

  for the case when we consider lossless networks

Confirms intuition that current flows can serve as proxies for power flows

Decoupled Power Flow: Connection to DC Power Flow

Flow on line (m, n) according to power divider

$$P_{(m,n)} = \alpha_{(m,n)}^{\mathrm{T}} P \tag{1}$$

Flow on line (m, n) according to DC power flow

$$P_{(m,n)} = -b_{mn}(\theta_m - \theta_n) \tag{2}$$

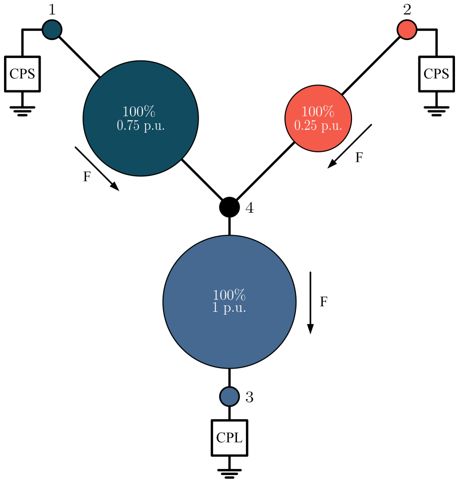
(1) boils down to (2) when

- Shunt susceptance terms are neglected
- (An admittedly artefactual) slack bus is assigned

Case Studies

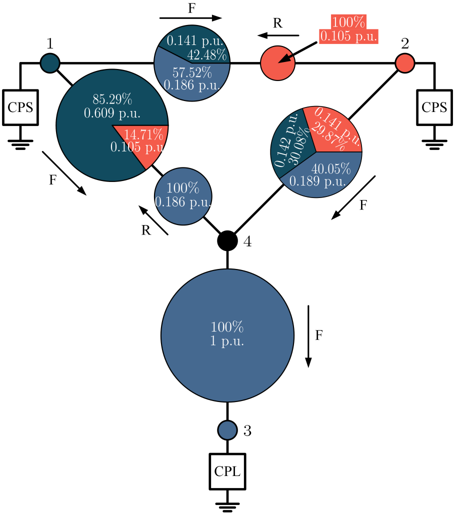
- Circulating power flow
- Transmission-network allocation
- Transmission-loss allocation

Case Study: Circulating Power Flow



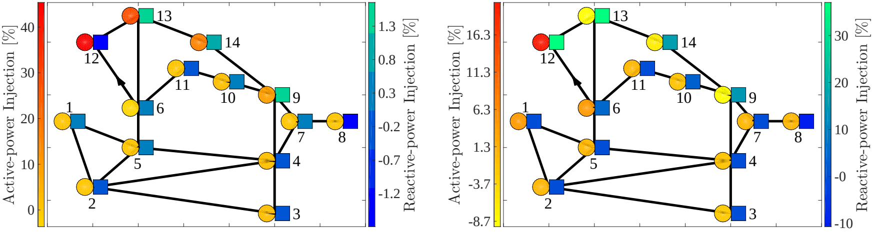
CPS: Constant Power Source, CPL: Constant Power Load, F: Forward, R: Reverse

Case Study: Circulating Power Flow



CPS: Constant Power Source, CPL: Constant Power Load, F: Forward, R: Reverse

Case Study: Transmission-network Allocation



$P_{(6,12)}$ $Q_{(6,12)}$

- Impact of bus active-power injections on line active-power flows is more dominant
- Buses 12 and 13 active-power injection contributions to $Q_{(6,12)}$ are 20.65% and -9.41%, respectively
- Both the active- and reactive-power injections in the network contribute to the reactive-power flow

Case Study: Transmission-loss Allocation

"[...] *system transmission losses are a nonseparable, nonlinear function of the bus power injections which makes it impossible to divide the system losses into the sum of terms, each one uniquely attributable to a generation or load.*"

Conejo, Galiana, and Kockar, "Z-bus loss allocation," TPWRS, 2001.

Losses on line (m, n) decomposed into nodal contributions

$$L_{(m,n)} = \Big(|V_m| u_{(m,n)}^{\mathrm{T}} + |V_n| u_{(n,m)}^{\mathrm{T}} \quad P + \Big(|V_m| v_{(m,n)}^{\mathrm{T}} + |V_n| v_{(n,m)}^{\mathrm{T}} \quad Q.$$

Case Study: Transmission-loss Allocation

Losses on line (m, n) decomposed into nodal contributions

$$L_{(m,n)} = \Bigg(|V_m| u_{(m,n)}^{\mathrm{T}} + |V_n| u_{(n,m)}^{\mathrm{T}} \quad {\color{red}P} + \Bigg(|V_m| v_{(m,n)}^{\mathrm{T}} + |V_n| v_{(n,m)}^{\mathrm{T}} \quad {\color{red}Q}.$$

Case Study: Transmission-loss Allocation

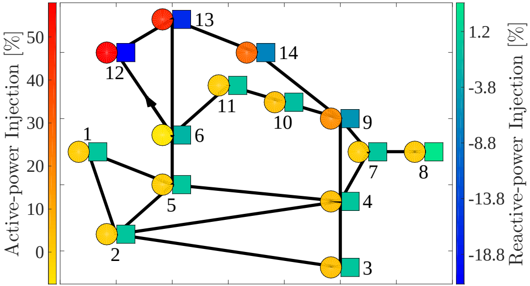
Losses on line (m,n) decomposed into nodal contributions

$$L_{(m,n)} = \Big(|V_m| u_{(m,n)}^{\mathrm{T}} + |V_n| u_{(n,m)}^{\mathrm{T}} \quad P + \Big(|V_m| v_{(m,n)}^{\mathrm{T}} + |V_n| v_{(n,m)}^{\mathrm{T}} \quad Q.$$

Sketch of derivation

$$L_{(m,n)} = P_{(m,n)} + P_{(n,m)}.$$

Case Study: Transmission-loss Allocation



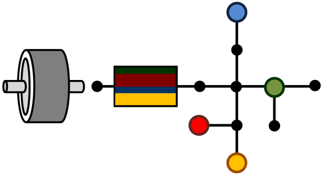
- Non-trivial impact of reactive-power injections on active-power losses
- Line $(6, 12)$: 27.4% of the loss from bus 14 active-power injection, and 16.8% from bus 13 reactive-power load

Summary

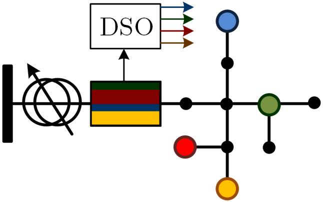
- Uncovered nonlinear mapping between injections and flows
- Connections to: i) Current divider, ii) DC power flow
- Applications to analysis of power networks

Future Work

- Obvious applications: flow control, pricing, visualization
- Power tracing: Uncover the fraction of active- and reactive-power generator outputs consumed by loads in the power network
- Operations and control: Realizing controllable aggregates of Distributed Energy Resources (DERs)



Power Tracing



Aggregating DERs

Questions?